b) This is the cyclic process which means the initial & the final state of the system is the same.

$\therefore \Delta U = 0$, $\therefore Q = W = -1000 J$.

ie 1000J of heat must be rejected by the system.

c) If the direction of the cycle is changed, the workdone will be +ve. The system will do work.

$\therefore$ The ~~8.~~ Total workdone in a cyclic process is +ve. if the process is goes around the cycle in a clockwise direction. The total workdone in a cyclic process is -ve, if the process goes around the cycle in anticlockwise direction.

CHAP:6:- SUPERPOSITION OF WAVES

Que:- Define Progressive wave and state its properties.

⇒ Progressive wave:- It is a wave which continuosly travel through the medium in a given direction, without any damping and obstruction, from one particle to another.

properties of progressive wave

1) Every particle in a medium executes the same type of vibrations & particles performing S.H.M.
2) All particles of medium have same amplitude, period and frequency.
3) The state of vibration (phase) changes from one particle to another
4) No particle remains permanently at rest. Each particle momentarily comes to rest at extreme position.
5) The velocity of particle is max^n when it pass through mean position.
6) During the propagation of wave there is transfer of energy not matter along the wave.
7) The velocity of ~~the~~ wave through the medium depends upon the properties of the medium.
8) Progressive waves are of two types (a) Transverse wave (b) Longitudinal wave.
9) In transverse wave vibrations of particles are $\perp$er to the direction of propagation of wave & produce crests and troughs in their medium of travel. In longitudinal

wave vibrations of particles are parallel to the direction of propagation & produces compressions & rarefactions along the direction of propagation.

ii) Both transverse and longitudinal waves can pass through solids, but only longitudinal wave can pass through Fluids (liquids & gases).

The eq^n of progressive wave travelling in +ve direction of x-axis is

$$\boxed{y(x,t) = A\sin(kx - \omega t)}$$

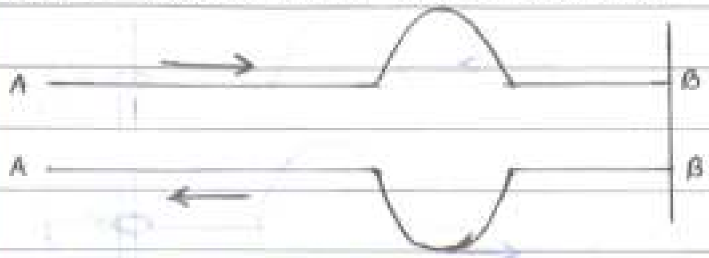
		Units
where A = Amplitude of wave,	–	metre
$k = \frac{2\pi}{\lambda}$ = wave number	–	rad/m
λ = wavelength	–	metre
ω = Angular frequency	–	rad/s
$V = \frac{\omega}{k}$ = speed of wave	–	m/s
$n = \frac{1}{T}$ = Freq^n of wave	–	Hz or s^{-1}.

The eq^n of progressive wave travelling in -ve direction of x-axis is

$$\boxed{y(x,t) = A\sin(kx + \omega t)}$$

6Que- Explain the reflection of transverse wave.

⟹ 1] Reflection of a wave pulse sent as a crest from a rarer medium to a denser medium.

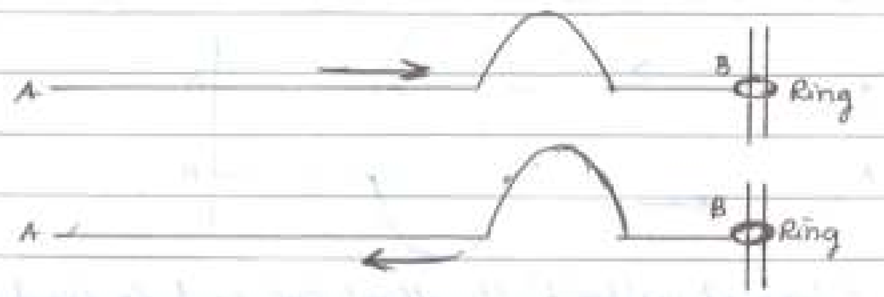


- A long string (light) AB attach one end to rigid support at B. (Hear string is rarer medium & rigid support as denser medium).
- A jerk giving at free end A, generate crest in the string.
- Observe what happens when this crest moves towards B?
- Observe what happen when the crest reaches B?
- Perform the same activity repeatedly & observe carefully.

Observation – It is observed that, when crest moves along the string towards B, it pulls the particles of string in upward direction. When it reach at B (rigid support), it tries to pull the point B upwards, but as it is rigid support, so B remains at rest and an equal and opposite reaction is produced on the string by Newton's 3rd law; So string is pulled downwards. So crest gets reflected as a trough or trough is reflected as crest.

Hence it is concluded that when transverse wave is reflected from rigid support, the crest is reflected as trough & vice versa. There is phase diff of π^c bet^n incident and reflected wave.

2] Reflection of a wave pulse sent as a crest from a denser medium to a rarer medium.



A long string AB attached at end B to a ring which can slide easily on a vertical metal rod without friction. (Hear string is as denser medium & end B attached to ring is at the reference of rarer medium)

- Give a jerk to free end A of the string
- Observe what happens when crest reaches the point B attached to the ring.
- Try to find the reason of the observed movement.

Observation - It is observed that when the crest reaches the point B, it pulls the ring upwards and the wave is seen to get reflected back as a crest. So crest is reflected as crest or trough is reflected as trough when transverse wave travel from denser to rarer medium.

There is no change of phase betn incident and reflected waves.

3> Reflection of crest from ⓐ Denser medium (heavy string) ⓑ rarer medium (light string).

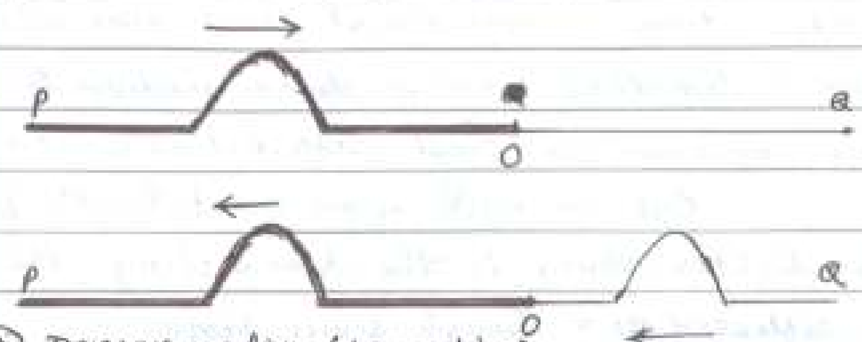


fig ⓐ Denser medium (heavy string)

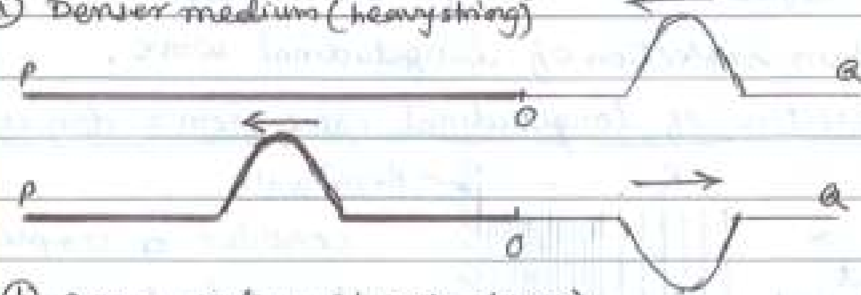


fig ⓑ Rarer medium (light string).

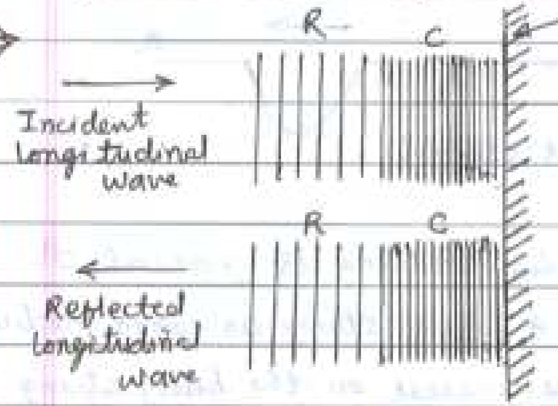
- A heavy string P & a light string Q join at O. (heavy string as denser & light string as rarer medium)
- produce a wave pulse as a crest on the heavy string P, moving towards O.
- observe the part of wave pulse reflected back on the heavy string.
- produce a wave pulse as a crest on a light string Q moving towards O.
- observe the part of wave pulse reflected on the light string Q.
- what diff is observed in two cases
- Try to find the reasons behind your observations.

Observation - we find that a crest travelling from the heavy string gets reflected as a crest from the lighter string i.e reflection at the surface when a wave is travelling from a denser medium to a rarer medium. i.e crest reflected as crest in fig (a)

But in fig (b) when crest travels from the lighter string to the heavy string, the crest is reflected as a trough & vice versa.

Que-* Explain reflection of longitudinal wave.

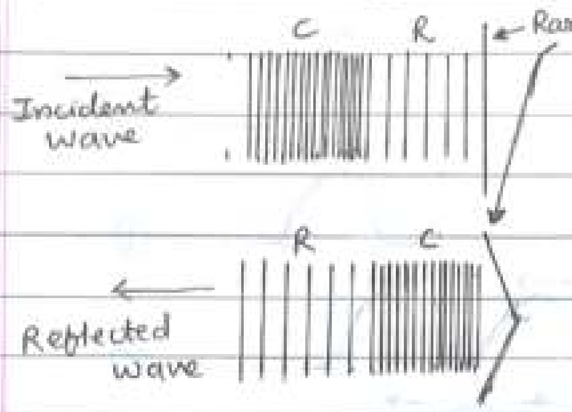
I) Reflection of longitudinal wave from a denser medium.



Consider a compression of longitudinal wave is incident on the denser surface (rigid wall), travelling from rarer medium. The compression is high pressure region. When it reaches the denser medium, it tries to push the particles of denser surface, but energy of particles in a rarer medium is not sufficient to compress the particles of denser medium. According to Newton's 3rd law, an equal & opposite reaction is exerted on the rarer particles, so rarer particles gets compressed. It shows that ~~when~~ compression is reflected as compression & rarefaction is reflected as rarefaction

when longitudinal wave travel from rarer medium to denser medium. Also there is Phase change of π^c [no change] of phase during the reflection.

II) Reflection of longitudinal wave from a rarer medium.



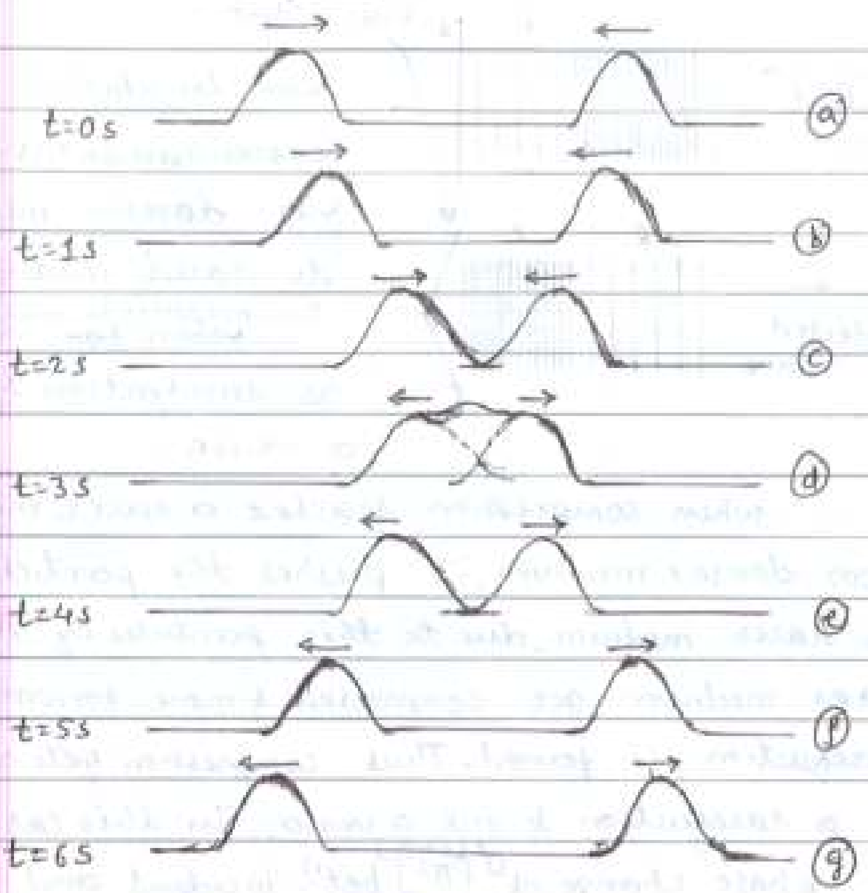
When longitudinal wave ~~Compress~~ (rarefaction) travel from denser medium to rarer medium, a compression is reflected as rarefaction & vice a versa.

When compression reaches a rarer medium from denser medium, it pushes the particles of the rarer medium, due to this particles of the rarer medium get compressed & move forward so rarefaction is formed. Thus compression gets reflected as a rarefaction & vice a versa. In this case there is phase change of 0 (zero) [π^c] bet^n incident and reflected wave.

* Principle of superposition of waves:-

When two or more waves, travelling through a medium pass through a common point, each wave produces its own displacement at that point, independent of the presence of other waves. The resultent displacement at that point is equal to the vector sum of the displacements due to the individual wave at that point.

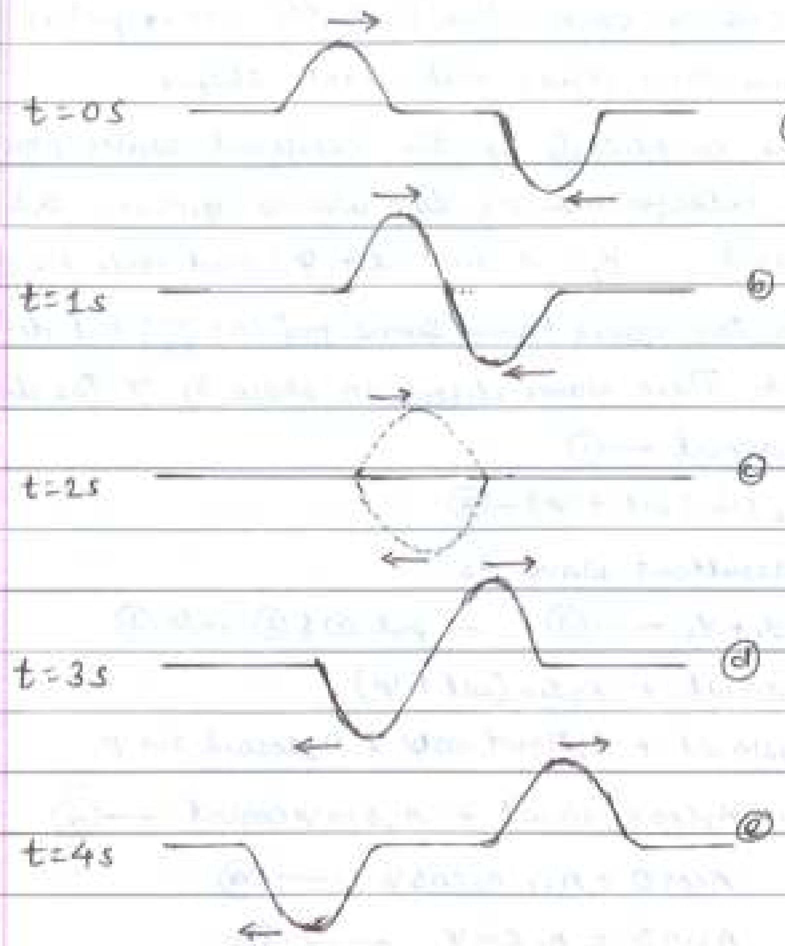
I] Superposition of two wave pulses of equal amplitude and same phase moving towards each other.



The propagation of approaching wave pulses, their superposition and their propagation after superposition are shown in figs. The waves cross each other bet^n $t=2s$ and $t=4s$, as shown in figs c, d & e. Hear the two waves superimpose the resultent displacement is equal to the sum of the displacements of individual wave pulses (dashed lines)

This is constructive interference. The displacement due to wave pulses after crossing at $t=5s$ & $t=6s$, both the waves continue to maintain their individual shapes.

II] Superposition of two wave pulses of equal amplitude and opposite phases moving towards each other:



The propagation of approaching wave pulses, their successive superposition after every second, their superposition and propagation are shown in figs.

These wave pulses superimpose at $t=2s$.

and the resultent displacement (Full lines) is zero, due to individual displacements (dashed lines) differing in phase exactly at 180°. This is destructive interference. Displacement due to one wave is cancelled by the other when they cross to each other as shown in fig ©. After crossing each other both the waves pulses continue and maintain their individual shapes.

Que - Find the amplitude of the resultent wave produced due to interference of two waves given as $y_1 = A_1 \sin \omega t$, $y_2 = A_2 \sin(\omega t + \psi)$ and state the diff cases.

⇒ consider two waves having same freq^n ($n = \frac{\omega}{2\pi}$) but diff amplitudes A_1 and A_2. These waves differ in phase by ψ. The displacement

$y_1 = A_1 \sin \omega t$ — ①

$y_2 = A_2 \sin(\omega t + \psi)$ — ②

The resultent wave is

$y = y_1 + y_2$ — ③ — put ① & ② into ③

$\therefore y = A_1 \sin \omega t + A_2 \sin(\omega t + \psi)$

$y = A_1 \sin \omega t + A_2 \sin \omega t \cos \psi + A_2 \cos \omega t \cdot \sin \psi$.

$y = (A_1 + A_2 \cos \psi) \sin \omega t + A_2 \sin \psi \cos \omega t$ — ④

Let $A \cos \theta = A_1 + A_2 \cos \psi$ — ⓐ

$A \sin \theta = A_2 \sin \psi$ — ⓑ

put ⓐ & ⓑ into ④

$\therefore y = A \cos \theta \cdot \sin \omega t + A \sin \theta \cdot \cos \omega t$

$\therefore y = A \sin(\omega t + \theta)$ — ⑤

Eq^n ⑤ is the resultent wave, have same freq. as that of interfering waves. A is the resultent amplitude.

* Amplitude of resultent SHM (A):

squaring and adding eq^{ns} ⓐ and ⓑ.

$A^2 \cos^2\theta + A^2 \sin^2\theta = (A_1 + A_2 \cos\psi)^2 + A_2^2 \sin^2\psi$.

$A^2 = A_1^2 + 2A_1A_2\cos\psi + A_2^2\cos^2\psi + A_2^2\sin^2\psi$.

$\therefore A^2 = A_1^2 + 2A_1A_2\cos\psi + A_2^2$.

$\therefore \boxed{A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\psi}}$ — ⑥

Special cases:

1) If the waves are in phase:

$\therefore \psi = 0$, $\cos\psi = \cos 0 = 1$.

$\therefore A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2} = \sqrt{(A_1+A_2)^2}$

$\therefore \boxed{A = (A_1 + A_2)}$

If $A_1 = A_2 = A$
The amplitude is max^m
$\therefore$ Amplitude $= 2A$.

2) If the waves are out of phase:

$\therefore \psi = \pi$, $\cos\pi = -1$.

$\therefore A = \sqrt{A_1^2 + A_2^2 - 2A_1A_2} = \sqrt{(A_1 - A_2)^2}$

$\therefore \boxed{A = (A_1 - A_2)}$

If $A_1 = A_2 = A$
The amplitude is min^m
$\therefore$ Amplitude $= 0$

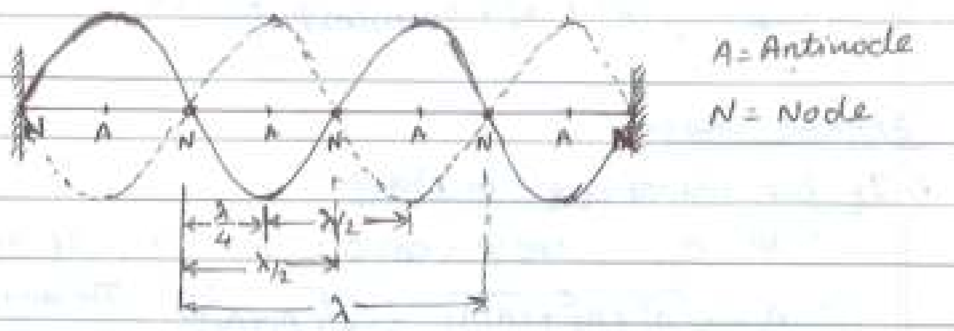
The Intensities of the waves proportional to square of their amplitudes, when $\psi = 0$

$\therefore I_{max} \propto (A_{max})^2 = (A_1 + A_2)^2$

when $\psi = \pi$

$\therefore I_{min} \propto (A_{min})^2 = (A_1 - A_2)^2$.

* Stationary wave: When two identical progressive waves travelling through the medium along the same path in exactly opposite direction interfere with each other & produce a resultent wave in the form of loops is known as stationary wave.



Que- Derive an expression for equation of stationary wave on a stretched string. State the conditions for node and antinode & hence show that nodes and antinodes are equally spaced.

⇒ Consider two S.H.P. waves of equal amplitude, period and wavelength travelling on a long string in opposite directions.

The wave travelling in +ve direction of x-axis is

$y_1 = a\sin 2\pi\left(nt - \frac{x}{\lambda}\right)$ — (1)

The wave travelling in -ve direction of x-axis is

$y_2 = a\sin 2\pi\left(nt + \frac{x}{\lambda}\right)$ — (2)

The resultent wave is

$y = y_1 + y_2$ — (3) -- Put (1) & (2) into (3)

$\therefore y = a\sin 2\pi\left(nt - \frac{x}{\lambda}\right) + a\sin 2\pi\left(nt + \frac{x}{\lambda}\right)$ — (4)

$\left[\because \sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cdot\cos\left(\frac{C-D}{2}\right)\right]$

$\therefore y = 2a\sin 2\pi nt \cdot \cos\frac{2\pi x}{\lambda}$ — (5) — $\left[\frac{n_1+n_2}{2} = n \quad \cos(-\theta) = \theta\right]$

Let $A = 2a\cos\frac{2\pi x}{\lambda}$

$\therefore \boxed{y = A\sin 2\pi nt}$ — (6)

Eq^n (6) is the eq^n of stationary wave of resultent amplitude A. It varies periodically with time in space.

* Condition for node:

Node: It is a point of stationary wave of minimum (zero) displacement.

$\therefore A = 0 \quad \left(\because A = 2a\cos\frac{2\pi x}{\lambda}\right)$

$\therefore 2a\cos\frac{2\pi x}{\lambda} = 0$ --- but $2a \neq 0$

$\therefore \cos\frac{2\pi x}{\lambda} = 0$

$\therefore \frac{2\pi x}{\lambda} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$ ----

$\therefore x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$ ---- (7)

Eq^n (7) represent the positions of nodes; it shows

The distance bet^n two successive nodes $= \frac{\lambda}{2}$.

The general equation is

$\boxed{x = (2P-1)\frac{\lambda}{4}}$ where P = 1, 2, 3 ----

* Condition for antinodes :-

Antinode :- It is a point of stationary wave of maximum displacement.

$\therefore A = max^m \quad \therefore A = \pm 2a \quad -- \left(\because A = 2a\cos\frac{2\pi x}{\lambda}\right)$

$\therefore 2a\cos\frac{2\pi x}{\lambda} = \pm 2a$

$\therefore \cos\frac{2\pi x}{\lambda} = \pm 1$

$\therefore \frac{2\pi x}{\lambda} = 0, \pi, 2\pi ------$

$\therefore x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2} ------ (8)$

Eq^n (8) represents the positions of antinodes, it shows

The distance bet^n two successive antinodes $= \frac{\lambda}{2}$

The general eq^n is

$\boxed{x = p\frac{\lambda}{2}} \quad -- p = 0, 1, 2 ------$

* Properties of stationary waves :-

1) stationary waves are produced due to superposition of two identical progressive waves (Transverse / Longitudinal) travelling through the medium along the same path in opposite directions.

2) If two identical transverse waves superimpose, the resultent wave is a transverse ~~wave~~ Stationary wave.

•) There are some points on the string which oscillate with min^m (zero) amplitude, are called as nodes

•) There are some points on the string which oscillate with maximum amplitude, are called as antinodes.

3) The points bet^n the nodes and antinodes vibrate with values of amplitude bet^n 0 and A.

4) If two identical longitudinal progressive waves superimpose, the resultent wave is a longitudinal stationary wave

5) The distance bet^n two successive nodes and successive antinodes is $\lambda/2$

6) The nodes and antinodes are alternately produced. The dist^n bet^n a node & adjacent antinode is $\lambda/4$.

7) The amplitude of vibration varies periodically in space. All points vibrate with the same frequency.

8) All the particles in the same loop are vibrate in same phase, while they are reverse in the adjacent loop.

Que - Distinguish bet^n Progressive wave and stationary wave

⟹

Progressive wave	stationary wave
1) The disturbance travel from one region to the other region with definite velocity	1) The disturbance remains in the region where it is produced. velocity of the wave is zero.
2) Amplitudes of all particles are same	2) Amplitudes of the particles are different.
3) The particles can not cross the mean position	3) All particles cross their mean positions simultaneously.
4) All particles are moving	4) Particles at ~~the~~ the position of nodes are always at rest.

5) Energy is transmitted from one region to another	5) There is no transfer of energy.
6) phases of adjacement particles are different	6) Betn two consecutive nodes the phase is same for all particles.

Que - Distinguish betn Free and forced vibrations.

⟹

Free vibrations	Forced vibrations
1) In this vibration the body vibrate with its natural frequency	1) In this vibration the body vibrate with the freqn of external force.
2) External periodic force is not required	2) External periodic force is required.
3) The body continuously loses energy	3) The body gains energy from external force.
4) The amplitude of vibrations of a body goes on decreasing	4) Amplitude of vibrations maintain by external force
5) Vibrations of body stops & it comes to rest	5) Body continuously vibrate & it does not stops.

* Harmonics :- It is the term used to denote fundamental freqn & all its integral multiple of freqns which are present or not in the given vibrations.

The lowest freqn of vibration is called as fundamental freqn or 1st harmonic.

- On the stretched string all harmonics are present.
- In a pipe open at both ends all harmonics are present.
- In a pipe closed at one end only odd harmonics are present.

* Overtones : It is the term used to denote the multiple of fundamental freqn which are actually present in the given vibrations.

- On the stretched string integral multiple of fundamental freqns are Overtones.
- In a pipe open at both ends integral multiple of fundamental freqns are overtones.
- In a pipe closed at one end only odd multiple of fundamental freqns are overtones.

* ~~End~~ End correction ($e = 0.3d$)

It is the distance betn the open end of a pipe and the actual position of antinode.

$\boxed{e = 0.3d}$ where d = Inner diameter of a pipe.

So the length of air column L & length of pipe l are diff.

$\boxed{L = l + e}$

ⓐ For a pipe closed at one end

The corrected length of an air column is

$\boxed{L = l + e}$

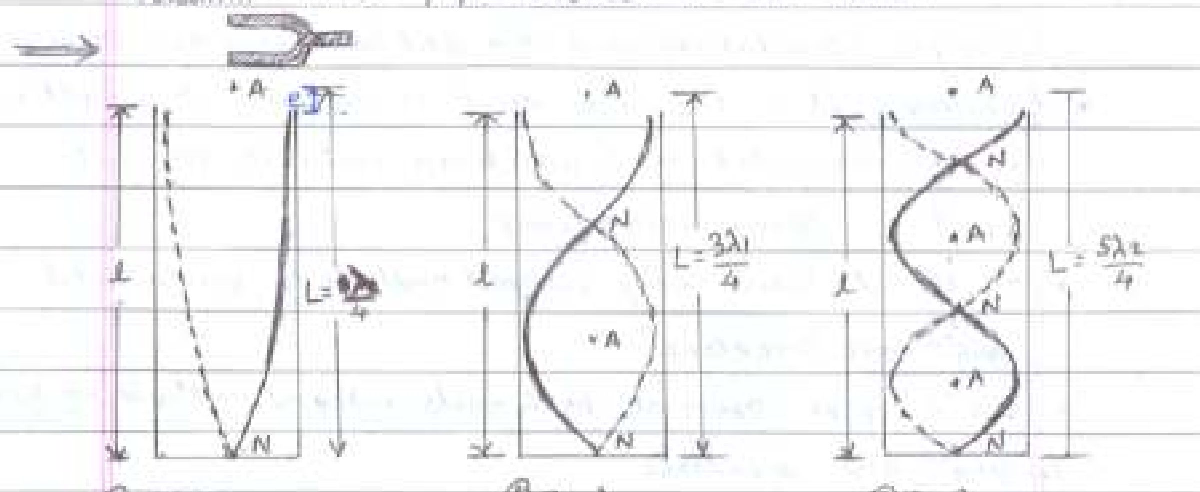
ⓑ For a pipe open at both ends

There are two end correction at both the open ends of a pipe, so the corrected length of an air column is

$\boxed{L = l + 2e}$

Que - Show that only an odd harmonics are present in an air column in a pipe closed at one end.

$\Longrightarrow$



ⓐ n, λ ⓑ n_1, λ_1 ⓒ n_2, λ_2

Consider a long cylindrical tube of length 'l' closed at one end. It consist an air column which is rigid at closed end & free to vibrate at the open end. When vibrating tuning fork is held at the open end, the longitudinal wave travel through air column & reflect from the closed end. The node is formed at the closed end & antinode is formed at the open end.

1) First mode (Fundamental mode): -

As shown in fig ⓐ an air column vibrate in the fundametal mode (1st mode) with freq^n n & wavelength λ. One node & one antinode is formed.

From fig ⓐ

$L = \frac{\lambda}{4}$

$\therefore \lambda = 4L$ — ①

we have

$v = n\lambda$ — ⓐ $\Rightarrow$ $n = \frac{v}{\lambda}$ — ②

put ① into ②

$\boxed{n = \frac{v}{4L} = \frac{v}{4(l+e)}}$ — Ⓐ

Eq^n Ⓐ represents Fundamental freq^n or 1st harmonic.

2) Second mode:

As shown in fig ⓑ an air column vibrate in second mode with freq^n n_1 & wavelength λ_1, with two nodes & two antinodes.

From fig ⓑ

$L = \frac{3\lambda_1}{4}$

$\therefore \lambda_1 = \frac{4L}{3}$ — ①

we have

$v = n_1\lambda_1$

$\therefore n_1 = \frac{v}{\lambda_1}$ — ②

put ① into ②

$n_1 = \frac{3v}{4L} = \frac{3v}{4(l+e)}$ — ③

From eq^n Ⓐ & ③

$\boxed{n_1 = 3n}$ — Ⓑ

Eq^n Ⓑ represents 3rd harmonic or 1st overtone.

3) Third mode:

As shown in fig ⓒ an air column vibrate in third mode with freq^n n_2 & wavelength λ_2, with three nodes & three antinodes.

From fig ©

$L = \frac{5\lambda_2}{4}$

$\therefore \lambda_2 = \frac{4L}{5}$ —— ①

we have

$v = n_2 \lambda_2$

$n_2 = \frac{v}{\lambda_2}$ —— ②

put ① into ②

$n_2 = \frac{5V}{4L} = \frac{5V}{4(l+e)}$ —— ③

From eqn Ⓐ & ③

$\boxed{n_2 = 5n}$ —— ©

Eqn © represents 5th harmonic or 2nd overtone.

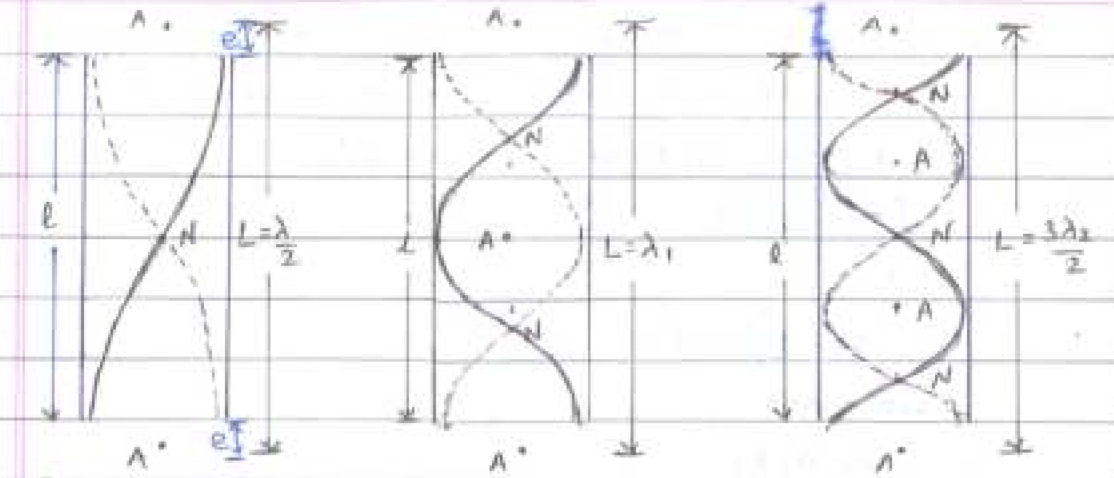
Eqns ⓐ, ⓑ & © shows that only odd harmonics are present.

For pth overtone, the freqn n_p is given by

$\boxed{n_p = (2p+1)n}$ where $p = 1, 2, 3$ ------

Que- show that all harmonics are present in an air column in a pipe open at both ends.

⟹ Consider a long cylindrical tube (pipe) of length 'l' open at both ends. It consist of an air column which is rigid at the centre & free to vibrate at the open ends. When vibrating tuning force is held above a open end, the longitudinal wave travel through air colum & reflect from the other end. A node is formed at the centre & antinode are formed at both the open ends.



ⓐ n, λ ⓑ n_1, λ_1 © n_2, λ_2.

1) First mode (Fundamental mode)

As shown in fig ⓐ an air column vibrate in the 1st mode (Fundametal mode) with freqn n and wavelength λ. one node and two antinodes are formed.

From fig ⓐ

$L = \frac{\lambda}{2}$

$\therefore \lambda = 2L$ —— ①

we have

$V = n\lambda$

$\therefore n = \frac{V}{\lambda}$ —— ②

put ① into ②

$\boxed{n = \frac{V}{2L} = \frac{V}{2(l+2e)}}$ —— Ⓐ

Eqn Ⓐ represents fundamental freqn or 1st harmonic.

2) Second mode :- As shown in fig (b) an air column vibrate in second mode with freq^n n_1 & wavelength λ_1. Two nodes and three antinodes are formed.

From fig (b)

$L = \frac{\cancel{2}\lambda_1}{\cancel{2}}$

$\therefore \lambda_1 = L$ — (1)

we have

$V = n_1 \lambda_1$

$\therefore n_1 = \frac{V}{\lambda_1}$ — (2)

put (1) into (2)

$\therefore n_1 = \frac{V}{L} = \frac{V}{(l+2e)}$ — (3)

From eq^n (A) & (3)

$\boxed{n_1 = 2n}$ — (B)

Eq^n (B) represents 2nd harmonic or 1st overtone.

3) Third mode : As shown in fig (c) an air column vibrate in third mode with freq^n n_2 and wavelength λ_2. Three nodes & four antinodes are formed.

From fig (c)

$L = \frac{3\lambda_2}{2}$

$\therefore \lambda_2 = \frac{2L}{3}$ — (1)

we have

$V = n_2 \lambda_2$

$\therefore n_2 = \frac{V}{\lambda_2}$ — (2)

put (1) into (2)

$n_2 = \frac{3V}{2L} = \frac{3V}{2(l+2e)}$ — (3)

From eq^n (A) & (3)

$\boxed{n_2 = 3n}$ — (C)

Eq^n (C) represents 3rd harmonic or 2nd overtone.

Eq^{ns} (A), (B) & (C) shows that all harmonics are present in an air column in a pipe open at both ends.

The freq^n of p^{th} overtone is given by.

$\boxed{n_p = (p+1)n}$ where $p = 1, 2, 3 - - - -$

* Practical determination of end correction :-

In this method use two pipes of same diameters but different lengths l_1 and l_2.

1) For a pipe closed at one end

$V = 4 n_1 L_1 = 4 n_2 L_2$

$\therefore n_1 L_1 = n_2 L_2$

$\therefore n_1(l_1+e) = n_2(l_2+e)$

$n_1 l_1 + n_1 e = n_2 l_2 + n_2 e$

$\therefore n_1 e - n_2 e = n_2 l_2 - n_1 l_1$

$(n_1 - n_2) e = n_2 l_2 - n_1 l_1$

$\therefore \boxed{e = \frac{n_2 l_2 - n_1 l_1}{n_1 - n_2}}$

2) For a pipe open at both ends:

$V = 2n_1L_1 = 2n_2L_2$

$\therefore n_1L_1 = n_2L_2$

$n_1(l_1+2e) = n_2(l_2+2e)$

$n_1l_1 + 2n_1e = n_2l_2 + 2n_2e$

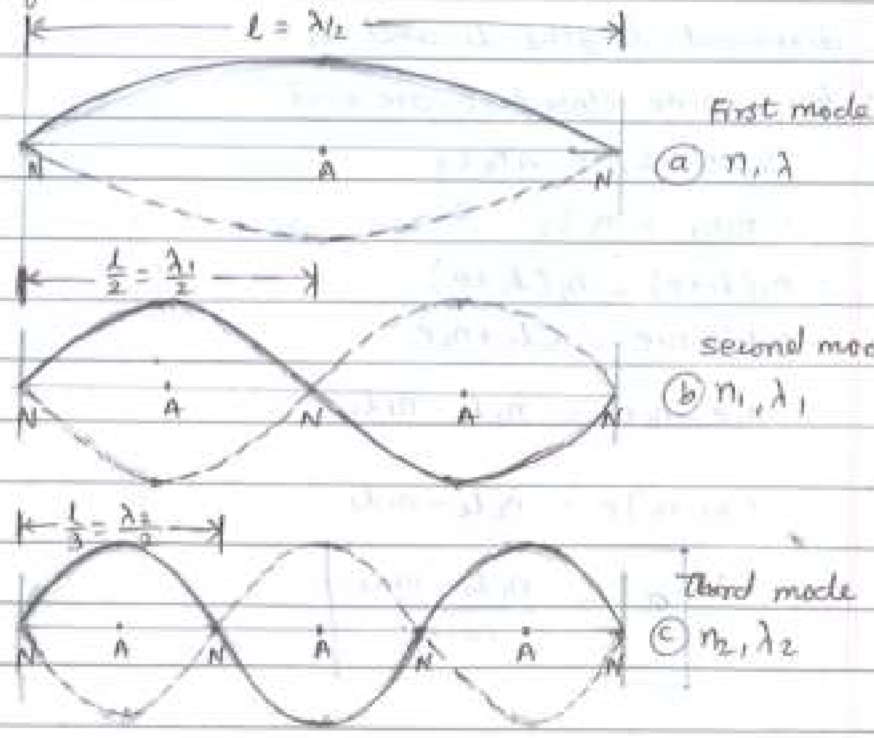
$2n_1e - 2n_2e = n_2l_2 - n_1l_1$

$2(n_1-n_2)e = n_2l_2 - n_1l_1$

$$\boxed{e = \frac{n_2l_2 - n_1l_1}{2(n_1-n_2)}}$$

Que- Explain the different modes of vibrations on a stretched string

$\Rightarrow$



A string of length 'l' & mass per unit length 'm' is stretched betn two rigid support.

1) First mode: The string is vibrated in 1st mode with freqn n & wavelength λ. One loop is formed with one antinode & two nodes. as shown in fig (a)

From fig (a) $l = \lambda/2$

$\therefore \lambda = 2l$ —— ①

we have $V = n\lambda$

$\therefore n = \frac{V}{\lambda}$ —— ②

put ① into ② & $V = \sqrt{\frac{T}{m}}$

$$\therefore \boxed{n = \frac{1}{2l}\sqrt{\frac{T}{m}}} \text{ —— (A)}$$

Eqn (A) represent fundamental freqn. or 1st harmonic.

2) Second mode:- If the string is prevented from vibrating by touching it with a light object at the centre & vibrate in 2nd mode with freqn n_1 & wavelength λ_1. Two loops are formed with two antinodes & three nodes. as shown in fig (b)

From fig (b)

$\frac{l}{2} = \frac{\lambda_1}{2}$

$\therefore \lambda_1 = l$ —— ①

we have $v = n_1\lambda_1$

$n_1 = \frac{V}{\lambda_1}$ —— ②

put ① into ② & $V = \sqrt{\frac{T}{m}}$

$\therefore n_1 = \frac{1}{l}\sqrt{\frac{T}{m}}$ — ③

From eqⁿ Ⓐ & ③

$\boxed{n_1 = 2n}$ — Ⓑ

Eqⁿ Ⓑ represents 2nd harmonic or 1st overtone.

3) Third mode:- The is vibrate in 3rd mode with freqⁿ n_2 & wavelength λ_2. Three loops are formed with three antinodes and four nodes as shown in fig Ⓒ

From fig Ⓒ

$\frac{l}{3} = \frac{\lambda_2}{2}$

$\lambda_2 = \frac{2l}{3}$ — ①

we have $v = n_2\lambda_2$

$\therefore n_2 = \frac{v}{\lambda_2}$ — ②

put into ② & $v = \sqrt{\frac{T}{m}}$

$\therefore n_2 = \frac{3}{2l}\sqrt{\frac{T}{m}}$ — ③

From eqⁿ Ⓐ & ③

$\boxed{n_2 = 3n}$ — Ⓒ

Eqⁿ Ⓒ represents 3rd harmonic or 2nd overtone.

Eqⁿˢ Ⓐ, Ⓑ & Ⓒ shows that all harmonics are present in ~~an air column for~~ the vibrations on the stretched string.

The freqⁿ of pth overtone is

$\boxed{n_p = (p+1)n}$ ---- $p = 1, 2, 3$ ----

Laws of Vibrating string:-

The fundamental freqⁿ of a vibrating string under tension is given by.

$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$

l = vibrating length of string
T = Tension applied to string
m = mass per unit length (linear density)

1) Law of length:- The fundamental freqⁿ of vibration of string is inversely proportional to its vibrating length if tension and mass per unit length are constant.

$n \propto \frac{1}{l}$ when T & m are constant

$\therefore nl = $ constant $\qquad \therefore \boxed{n_1 l_1 = n_2 l_2}$

2) Law of tension:- The fundamental freqⁿ of vibration of string is directly proportional to the square root of the tension applied to the string, if vibrating length and mass per unit length are constant.

$\therefore n \propto \sqrt{T}$ when l & m are constant

$\therefore \frac{n}{\sqrt{T}} = $ constant $\qquad \therefore \frac{n_1}{\sqrt{T_1}} = \frac{n_2}{\sqrt{T_2}} \qquad \therefore \boxed{\frac{n_1}{n_2} = \sqrt{\frac{T_1}{T_2}}}$

3) Law of linear density: The fundamental freqⁿ of vibration of string is inversely proportional to the square root of mass per unit length of string, if vibrating length & tension applied to string are constant.

$n \propto \frac{1}{\sqrt{m}}$ when l & T are constant.

If r is radius & ρ is density of material of string, The linear density is given by

linear density = mass per unit length = $\frac{\text{Vol}^m \text{ per unit}}{\text{length}}$ x density

$$m = \frac{\pi r^2 l}{l} \times \rho$$

$$m = \pi r^2 \rho$$

$$\therefore n \propto \frac{1}{\sqrt{\pi r^2 \rho}}$$

ie $n \propto \frac{1}{\sqrt{\rho}}$ and $n \propto \frac{1}{r}$

Que- Describe the verification of laws of a vibrating string.

⟹ I] Verification of First law of vibrating string:

Measure the mass M & length L of wire & determine its mass per unit length ($m = M/L$). Stretched the wire on a sonometer box & apply suitable tension T to the wire. put two knife edges (bridges) below the wire & a light paper rider on the wire midway betn the knife edges.

Put a vibrating tuning fork of freqn n, on the sonometer box & adjust the distance betn knife edges, the natural freqn of wire changes. At a particular length l_1 the wire vibrate with maxm amplitude & paper rider will thrown off. Now use a set of tuning forks of freqns n_1, n_2, n_3 --- & measure the corresponding vibrating lengths. l_1, l_2, l_3 --- It is found that $n_1 l_1 = n_2 l_2 = n_3 l_3 = \ldots = $ constant for constant value of tension (T) & mass per unit length (m)

$\therefore nl =$ constant if T & m are constant

ie $\boxed{n \propto \frac{1}{l}}$ at T & m constant.

II] Verification of second law of vibrating string:

The vibrating length 'l' & mass per unit length of the given wire are kept constant. By changing the tension the same length 'l' vibrate in unison with diff tuning forks of diff freqns n_1, n_2, n_3 --- for which the respective tensions are T_1, T_2, T_3 ----. It is observe that

$$\frac{n_1}{\sqrt{T_1}} = \frac{n_2}{\sqrt{T_2}} = \frac{n_3}{\sqrt{T_3}} = \text{---} = \text{constant}$$

$$\therefore \frac{n}{\sqrt{T}} = \text{constant}$$

$\therefore \boxed{n \propto \sqrt{T}}$ at l & m are constant

III] Verification of third law of vibrating string:

Two wires of diff. mass per unit lengths are used. (m_1 & m_2). The first wire is subjected to suitable tension & made to vibrate in unison with a given tuning fork. The vibrating length is l_1. Using same fork, under same tension, the second wire is made to vibrate, the vibrating length is l_2. As freqn n & tension T are fixed. It is observed that

$$l_1 \sqrt{m_1} = l_2 \sqrt{m_2}$$

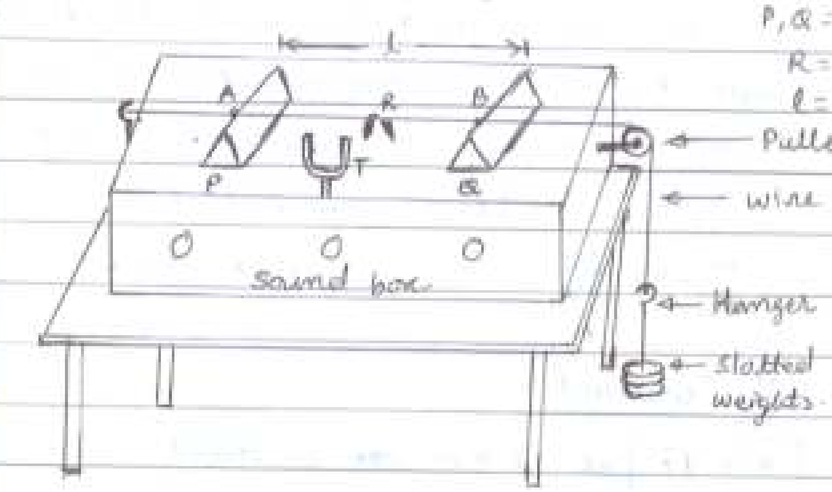
$$\therefore l\sqrt{m} = \text{constant}$$

But by 1st law of a vibrating string $n \propto \frac{1}{l}$.

$\therefore \boxed{n \propto \frac{1}{\sqrt{m}}}$ if T & l are constant

Que- Describe Sonometer experiment to identify the resonance with neat labelled diagram.

⟹



P, Q = Knife edges
R = Paper rider
l = vibrating length.

Construction: A sonometer consist of a hollow rectangular wooden box (sound box). Two bridges P & Q are placed along the width of box, which can slide (move) along its length. A metal wire of uniform cross section is fixed at one end & passes over bridges & pulley & attach to the hanger with suitable slotted weights. By changing the weights, the tension in the wire can change. If the wire is plucked at a midway bet^n the bridges, a transverse wave is produced in the wire & so stationary wave is produced bet^n the bridges due to reflection & their superposition.

Working: Put a light paper rider on the string midway bet^n the bridges & a vibrating tuning fork is held near the wire, due to forced vibrations

the wire vibrates with the freq^n of tuning fork. Change the distance bet^n P & Q at points A & B the natural freq^n of the string becomes equal to the freq^n of tuning fork, so due to resonance the wire vibrates with max^m amplitude because the antinode is formed at midway of A & B so the paper rider flies off the wire. Measure the vibrating length 'l', mass per unit length m of wire & applied tension T. the freq^n of tuning fork is given by

$$\boxed{n = \frac{1}{2l}\sqrt{\frac{T}{m}}}$$

* Formation of beat:-

It is the phenomenon of superposition of two sound waves having same amplitude but slightly diff. freq^{ns} travelling in the same direction, the intensity of sound varies periodically with time, is known as formation of beat.

* Waxing: It is the max^m intensity of sound produced due to superposition of two sound waves having same amplitude but slightly diff in freq^{ns}
* Waning: It is the min^m intensity of sound produced due to superposition of two sound waves having same amplitude but slightly diff in freq^{ns}
* One waxing & successive waning forms one beat
* The number of beats per sec. is called as freq^n of beat
* The time interval bet^n two successive beats is called as Period of beat

Que: Describe analytical method to determine beat frequency. Show that beat freq^n is equal to the freq diff of freq^ns of two sound waves ($N = n_1 - n_2$). For waxing and waning.

$\Rightarrow$ Consider two sound waves having same amplitude 'a' & slightly diff in freq^n n_1 & n_2. They arrive at a point in phase at point x of the medium. The displacement of these waves are

$$y_1 = a\sin 2\pi\left(n_1 t - \frac{x}{\lambda_1}\right)$$

$$y_2 = a\sin 2\pi\left(n_2 t - \frac{x}{\lambda_2}\right)$$

Let us assume that listner is at $x = 0$.

$$\therefore y_1 = a\sin 2\pi n_1 t \quad \text{--- (1)}$$

$$\therefore y_2 = a\sin 2\pi n_2 t \quad \text{--- (2)}$$

The resultent wave is

$$y = y_1 + y_2 \quad \text{--- (3)}$$

put (1) & (2) into (3)

$$y = a[\sin 2\pi n_1 t + \sin 2\pi n_2 t] \quad \text{--- (4)}$$

$$\left[\because \sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cdot\cos\left(\frac{C-D}{2}\right)\right]$$

$$\therefore y = 2a\sin\left[2\pi\left(\frac{n_1+n_2}{2}\right)t\right]\cdot\cos\left[2\pi\left(\frac{n_1-n_2}{2}\right)t\right]$$

Let $n = \frac{n_1+n_2}{2}$, $A = 2a\cos 2\pi\left(\frac{n_1-n_2}{2}\right)t$

$$\therefore \boxed{y = A\sin 2\pi n t} \quad \text{--- (5)}$$

Eq^n (5) is resultent progressive wave having freq^n n & amplitude A varies periodically with time.

The amplitude of sound is proportional to the square of amplitude. So resultent intensity is max^m at max^m amplitude

* Waxing (For max^m amplitude)

$\therefore A = \pm 2a \qquad \because 2a\cos 2\pi\left(\frac{n_1-n_2}{2}\right)t = \pm 2a$

$$\therefore \cos 2\pi\left(\frac{n_1-n_2}{2}\right)t = \pm 1$$

$$\therefore 2\pi\left(\frac{n_1-n_2}{2}\right)t = 0,\ \pi,\ 2\pi,\ 3\pi\ -----$$

$$\therefore t = 0,\ \frac{1}{n_1-n_2},\ \frac{2}{n_1-n_2},\ --------- \quad (6)$$

Eq^n (6) represents the positions of waxings.

$\therefore$ Time interval bet^n two successive waxings is period of beat

$$T = \frac{1}{n_1-n_2} \longrightarrow \text{Period of beat}$$

The freq^n of beat is $N = \frac{1}{T}$.

$$\therefore \boxed{N = n_1 - n_2} \text{ --- (A)}$$

* Waning (For min^m amplitude)

$\therefore A = 0 \qquad \therefore 2a\cos 2\pi\left(\frac{n_1-n_2}{2}\right)t = 0 \quad (\because 2a \neq 0)$

$$\therefore \cos 2\pi\left(\frac{n_1-n_2}{2}\right)t = 0$$

$$\therefore 2\pi\left(\frac{n_1-n_2}{2}\right)t = \frac{\pi}{2},\ \frac{3\pi}{2},\ \frac{5\pi}{2}\ -----$$

$$\therefore t = \frac{1}{2(n_1-n_2)},\ \frac{3}{2(n_1-n_2)},\ \frac{5}{2(n_1-n_2)}\ ------ \quad (7)$$

Eq^n (7) represents positions of wanings

The time interval bet^n two successive wanings is period of beat

$$\therefore T = \frac{1}{n_1-n_2} \longrightarrow \text{Period of beat}$$

The freq^n of beat is $N = \frac{1}{T}$

$$\therefore \boxed{N = n_1 - n_2} \text{ --- (B)}$$

* Applications of beats

1) The phenomenon of beats is used for matching the freq^ns of diff. musical instruments by artists. They go on tuning until no beats are heard by their sensitive ers. ie the freq^ns of the instruments are identical & their effect gives a pleasant music.

2) The speed of aeroplane can be determined by using Doppler RADAR

When the source of sound and listner (or any one) is moving with respect to air (relative motion), the listner detects a sound whose freq^n is different from the freq^n of the sound source (apparent freq^n). This is known as Doppler effect.

A microwave (radio waves) signal of known freq^n is sent towards the moving aeroplane, it gets reflect from the aeroplane & received back at the microwave source. Due to the phenomenon of beats we can calculate the velocity of the aeroplane.

3) Unknown freq^n of a sound note can be determined by using the phenomenon of beats, using a known freq^n of sound note.

* characteristics of sound

1) Loudness
2) Pitch
3) Quality or timbre.

* **Loudness:-** Loudness is the human perception to intensity of sound. When sound travel through medium compressions & rarefactions are produced due to change in pressure. When sound is heard by a human, the wave exerts pressure on the human ear. The pressure variation is related to the amplitude & hence to the intensity. Where intensity is directly proportional to the square of amplitude. The variation in pressure from 28 Pa for the loudest tolerable sound to 2×10^{-5} Pa for the feeblest sound (whisper)

The response of human ear to sound is exponential and not linear, it depends amount of sound energy crossing per unit area around a point per unit time.

Scientifically, sound is specified not by its intensity but by the sound level β in decibles (dB)

$\beta = 10 \log \frac{I}{I_0}$ where I_0 min^m reference level of intensity ($10^{-12}\,W/m^2$) that normal human ear can hear. When $I = I_0$, $\beta = 0$ ie standard reference intensity has measure of sound level 0 dB. The unit of diff. in loudness is bel. 1 decibel $= \frac{1}{10}$ bel.

Min^m audible sound is 0 dB while whispering is 10 dB and normal speech have levels 60 dB at a dist^n of 1m from the source. The max^m tolerable sound is 120 dB.

Loudness is diff at diff freq^ns even for same intensity. For measuring loudness the unit is Phon.

Phon is a measure of loudness. It is equal to the loudness in decibel of any equally loud pure tone of freq^n 1000 Hz.

2) **Pitch**:- Pitch is the human perception to freq^n. High freq^n denotes higher pitch. The pitch of a female voice is higher than that of male voice [Female - shrill, Male - Flat]
The sensation of sound which helps the listner to distinguish bet^n a high freq^n & a low freq^n note.

3) **Quality or timbre**:- It is the characteristics of of sound which enables us to distinguish bet^n two sounds of same pitch and loudness we can detect the voice of a person or an instrument due to its quality. It depends on no. of overtones present in the sound along with a given freq^{n}. The sound generated by a source has a no. of freq^n components with diff. amplitudes.

* **Musical scale**:- It is a sequence of freq^{ns} which have a specific relationship with each other.
Normally in Indian classical music and western classical music eight freq^{ns} are in the specific ratio form an octave. Each freq^n gives a specific note. In a given octave freq^n increases along sa, re, ga, ma, pa, dha, ni, sa. (as well as Do, Re, Mi, Fa, So, La, Ti, Do).
The values of freq^{ns} are 240, 270, 300, 320, 360, 400, 450, 480 Hz respectively.

* **Musical Instruments**:-

A) Stringed instrument:- It consist of stretched strings. Sound is produced by plucking of strings. By adjusting tension in the string, they can tuned to certain freq^{ns}.

1) Plucked string type:- In these instruments string is plucked by fingers ~~eg.~~
eg. Tanpura, sitar, guitar, veena etc

2) Bowed string type:- In these instruments a string is played by bowing
eg. violin, sarangi, etc

3) Struck string type:- In these instruments a string is struck by a stick
eg. santoor, piano. etc

B) Wind instrument:- These instruments consist of air column. Sound is produced by setting vibrations of air column.

1) Freewind type: In these instruments free brass reeds are vibrated by air. The air is either blown or compressed
eg. mouth organ, harmonium etc

2) Edge type: In these instruments air is blown against an edge
eg. Flute

3) Reedpipes: It consist of single or double reeds & also instruments without reeds.
eg. Saxophone, clarinet (single reed); bassoon (double reed)
bugle (without reed)

c] Percussion Instruments :-

In these instruments sound is produced by setting vibrations in stretched elastic membrane eg. Tabla, drum, dhol, mridangam, sambal, daphali etc.

They also consist of metal type of instruments which produce sound ~~by~~ when they struck against each other or with a beater

eg. cymbals (jhanja), xylophone etc.